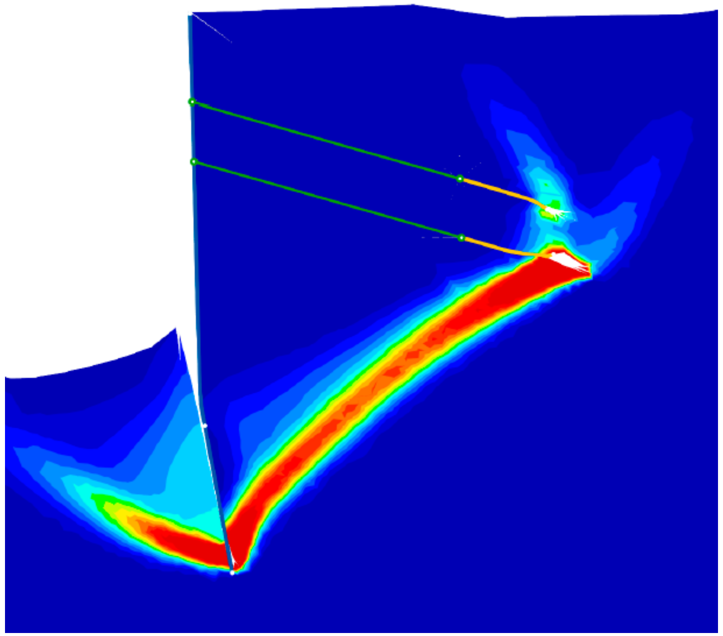


Optum CE

Geomechanics

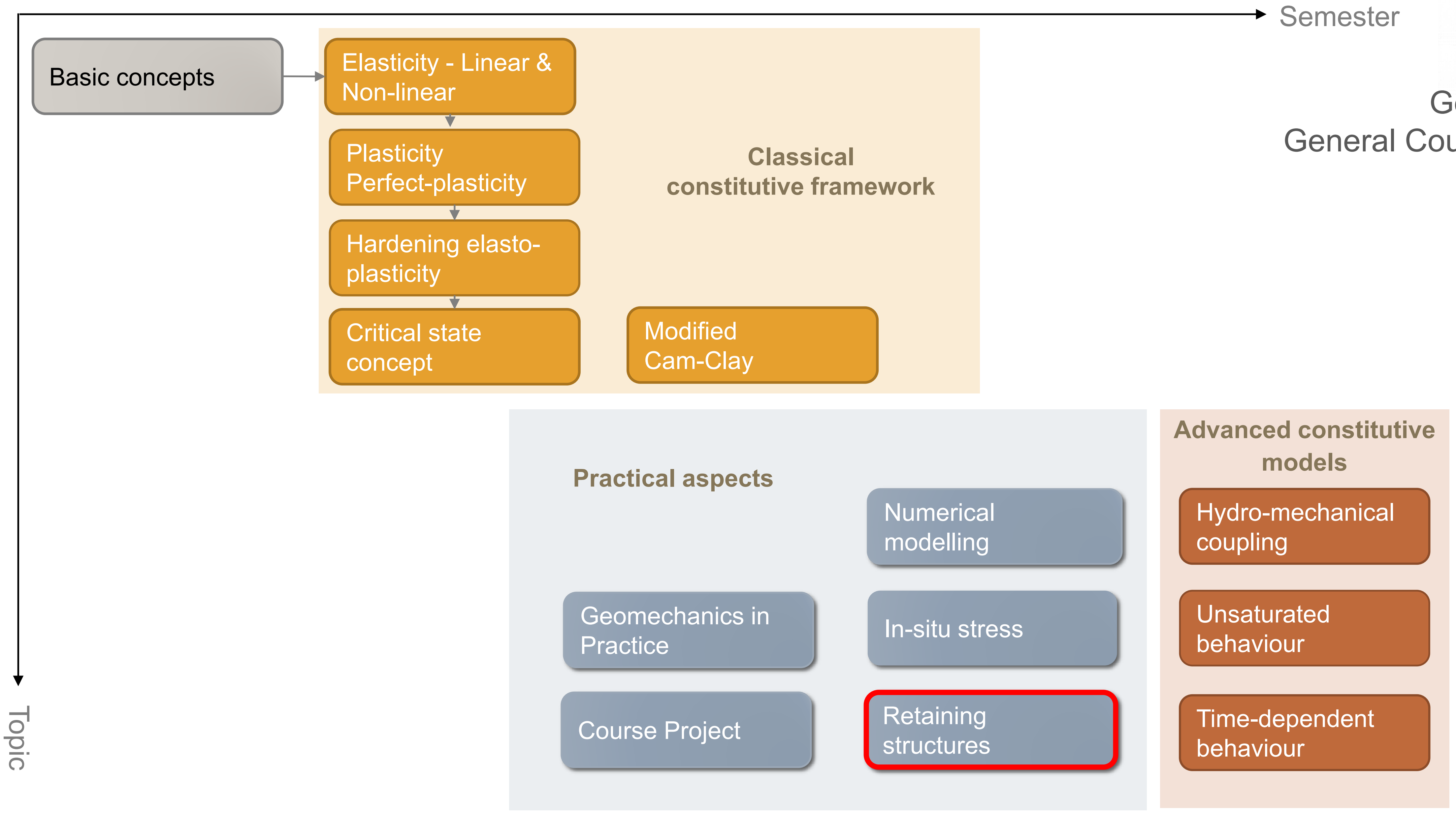
LECTURE 12

RETAINING STRUCTURES

DR. ALESSIO FERRARI

Laboratory of soil mechanics – Fall 2025

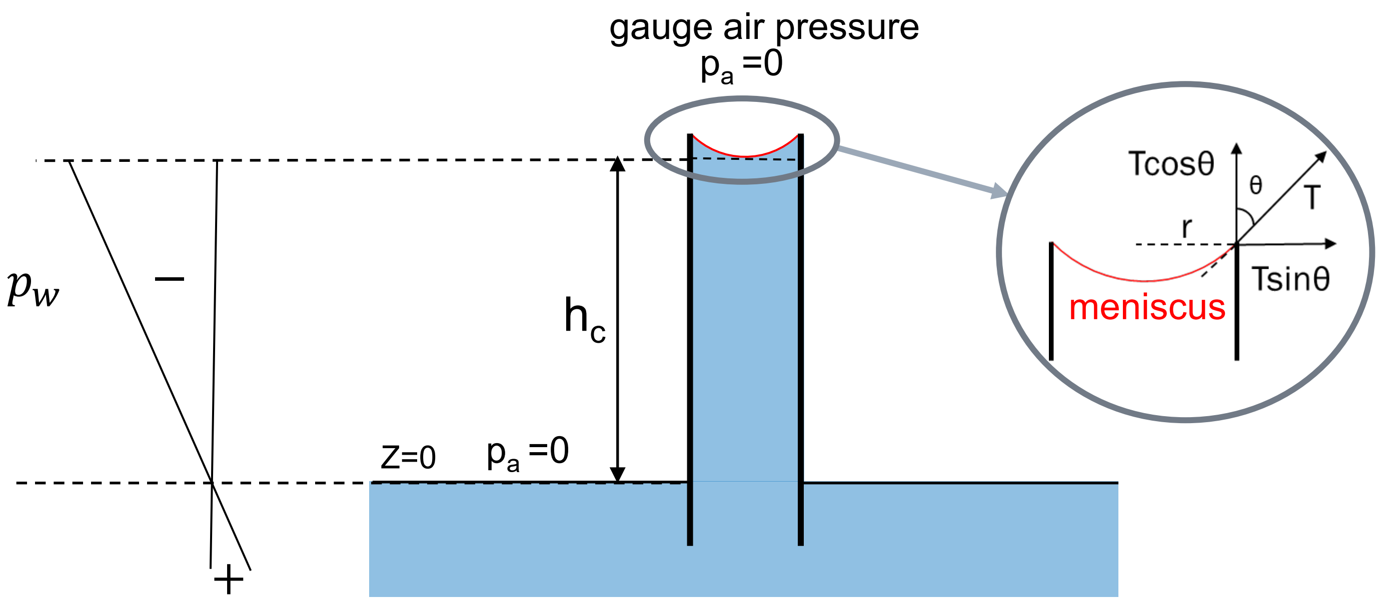
Geomechanics
General Course overview



Recall – Unsaturated soils

Complex interaction between water, air and solid phases

- The interface air-water near a solid surface has a curvature due to **surface tension**
- The water in the voids is similar to the water in a capillary tube.



T: Surface tension (force per unit of length)
θ: contact angle (it depends on the characteristics of the liquid and the solid)

Vertical equilibrium of the water column

$$\gamma_w h_c \pi r^2 = 2\pi r T\cos\theta$$

$$h_c = \frac{2\pi r T\cos\theta}{\gamma_w \pi r^2} = \frac{2T\cos\theta}{\gamma_w r}$$

$$p_w = -\gamma_w h_c = -\frac{2T\cos\theta}{r}$$

The gauge water pressure is negative (p_w)

Young-Laplace equation

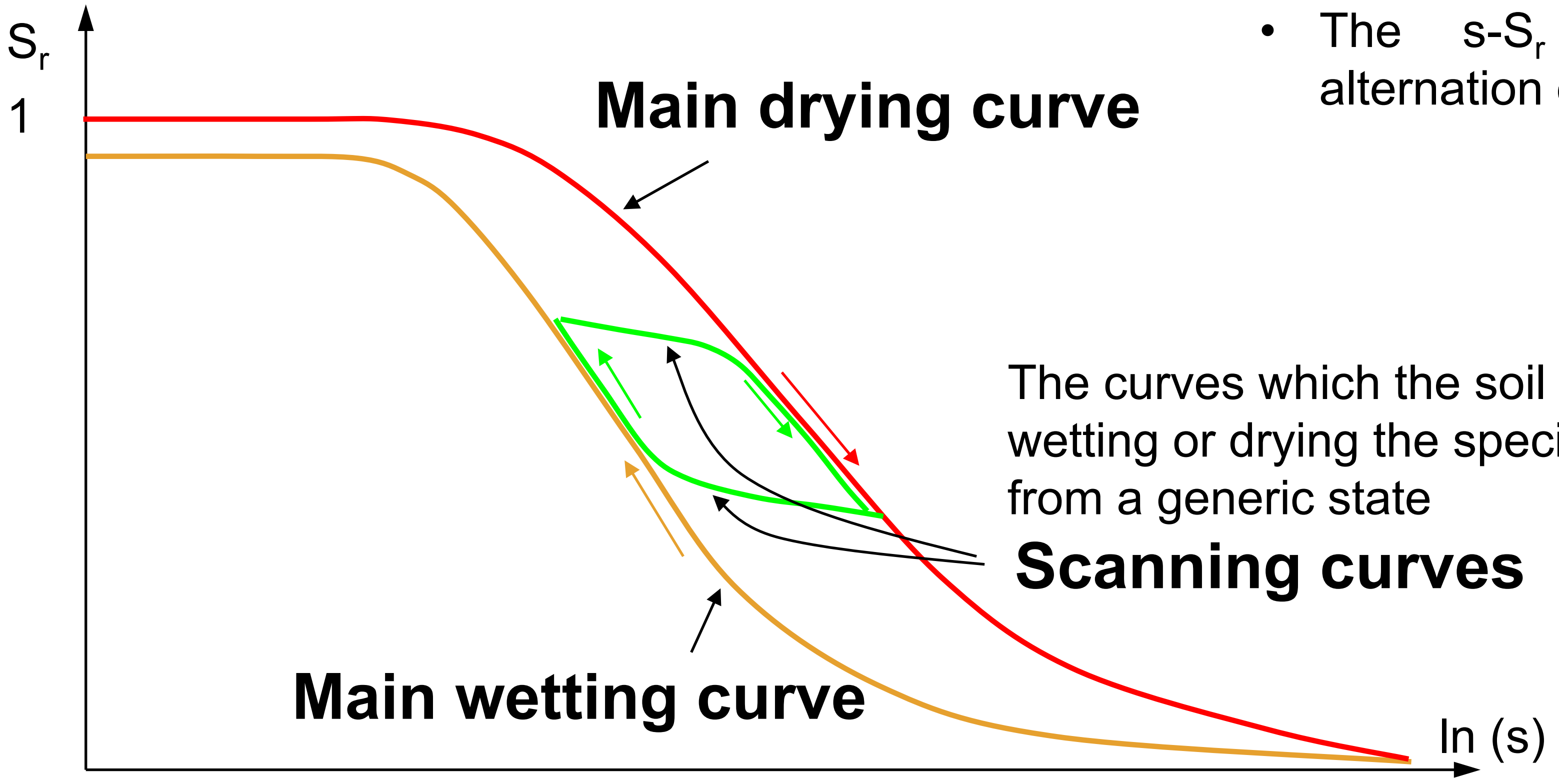
$$s = p_a - p_w = \frac{4T\cos\theta}{D}$$

Suction s

$$D = 2r$$

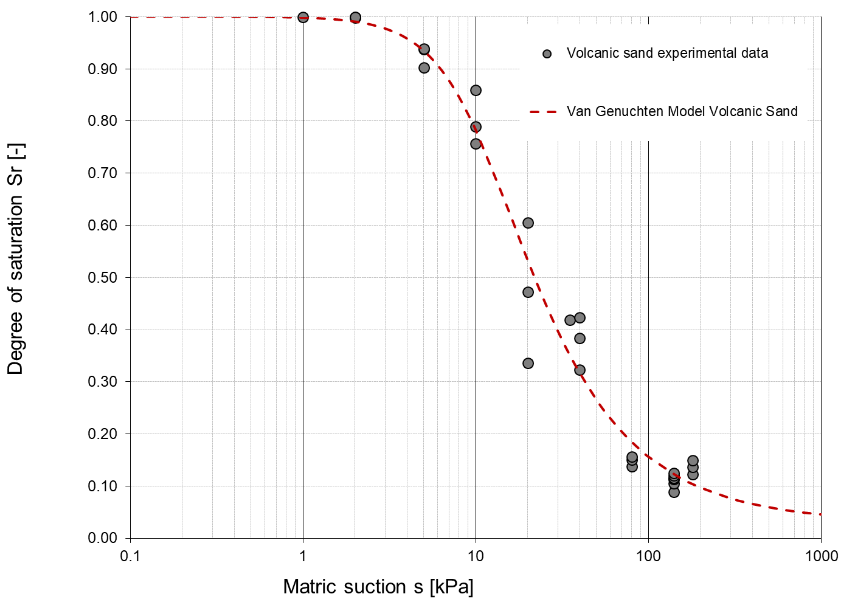
Recall - WRC

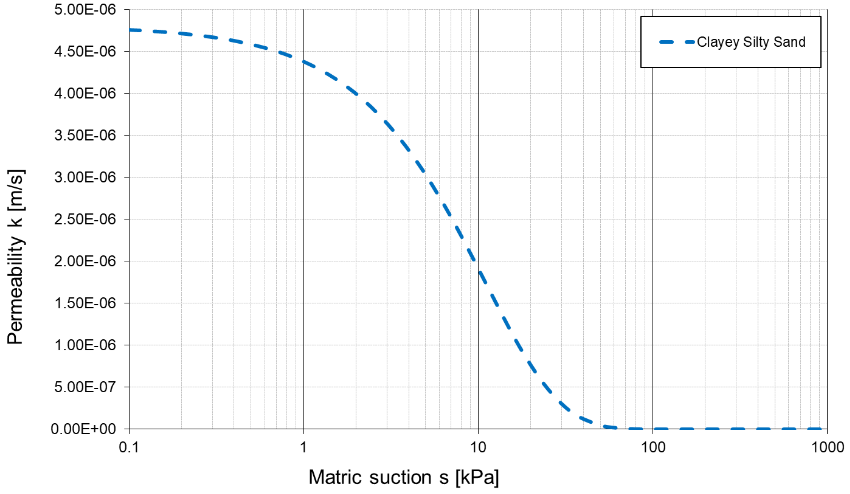
DRYING AND WETTING PROCESSES – HYSTERETIC BEHAVIOR



- Increase in suction is called “drying process”
- Decrease in suction is called “wetting process”
- The s-S_r relationship depends on the alternation of these processes

Recall - WRC and permeability





Van Genuchten, M., 1980

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

α ,n, m fitting parameters

The evolution of the coefficient of permeability with suction can be described for example by using the Gardner's model (1958).

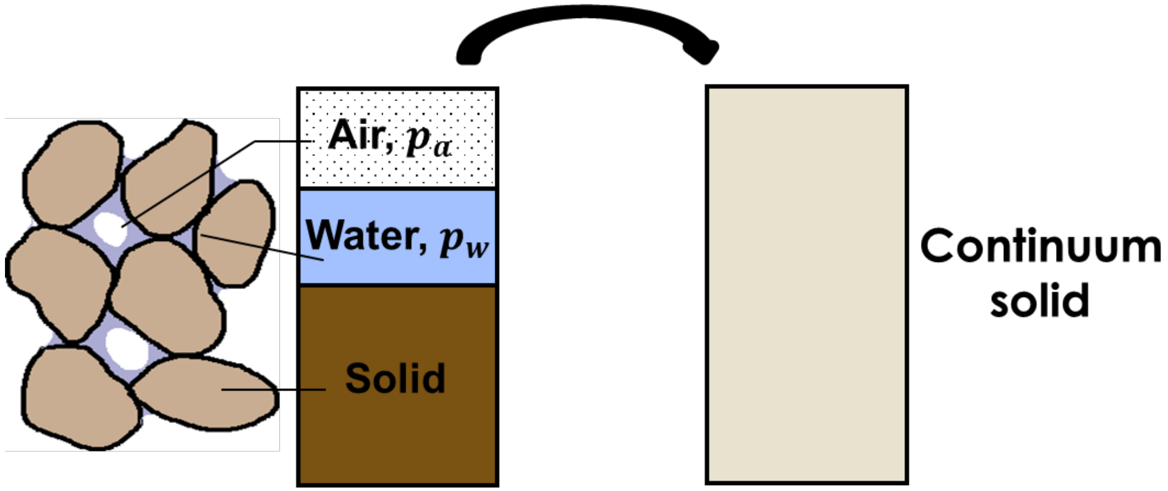
Gardner's model

$$k = k_{sat} e^{-\alpha(p_a - p_w)}$$

α fitting parameter

Recall - Effective stress

The effective stress in the specific case of unsaturated soils



Multi-phase description

Single-phase description

$$\sigma'_{ij} = \sigma_{ij} - \sum_{\beta=1}^{2} \alpha_\beta p_\beta \delta_{ij}$$

$$\sigma'_{ij} = \sigma_{ij} - (1-\chi)p_a\delta_{ij} - \chi p_w \delta_{ij}$$

$$\sigma'_{ij} = (\sigma_{ij} - p_a\delta_{ij}) + \chi(p_a - p_w)\,\delta_{ij}$$

$$\sigma'_{ij} = \sigma_{net,ij} + \chi s \delta_{ij}$$

Net stress tensor ($\sigma_{net,ij}$) **Suction stress tensor** ($\chi s \delta_{ij}$)

with $\chi = f(S_r)$

if χ = Sr $\quad \sigma'_{ij} = \sigma_{net,ij} + Sr \cdot s\delta_{ij}$

Outline

- Lateral earth pressure profiles for dry and saturated soils at rest, at active and at passive state
- Lateral earth pressure profiles for unsaturated soils
  - Shear strength of unsaturated soils
- Vertical trench - Critical height
- Effects of infiltrations on the lateral earth pressure
  - Darcy’s law for unsaturated soils
- Conclusions

Retaining structures

Retaining structures are built in order to provide a SUPPORT ACTION against the LATERAL EARTH PRESSURE of soils, distinguished as:

- On-site soils, whose initial equilibrium conditions are modified as a result of excavation operations

- Transported soils employed in the construction of geo-structures like embankments or bridge abutments.

Typologies of retaining structures

Cantilevered Retaining Walls

Embedded Retaining Walls

Gravity Retaining Wall

Crib Walls

Gabions

Reinforced Soil Slopes

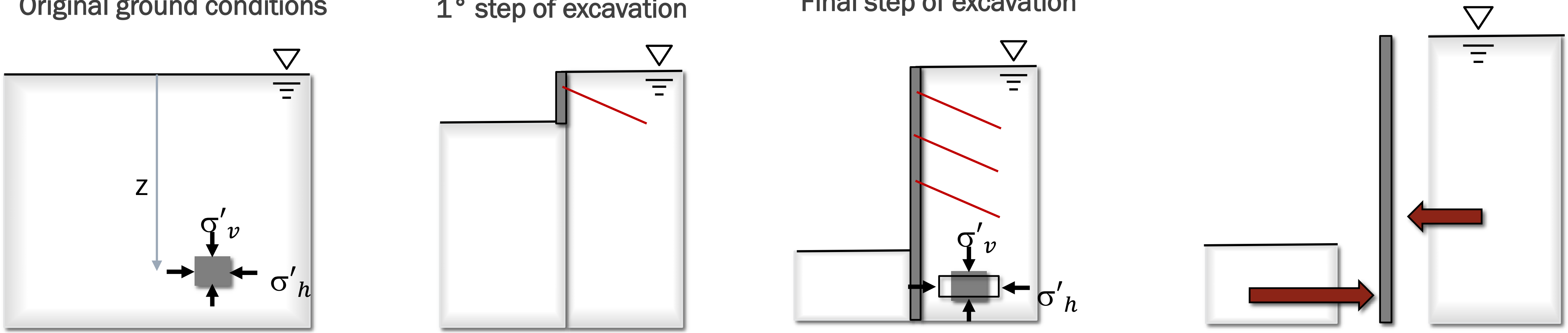
Lateral Earth Pressure

How to evaluate the Lateral Earth pressure of soils?

Original ground conditions

1° step of excavation

Final step of excavation



Rankine's theory

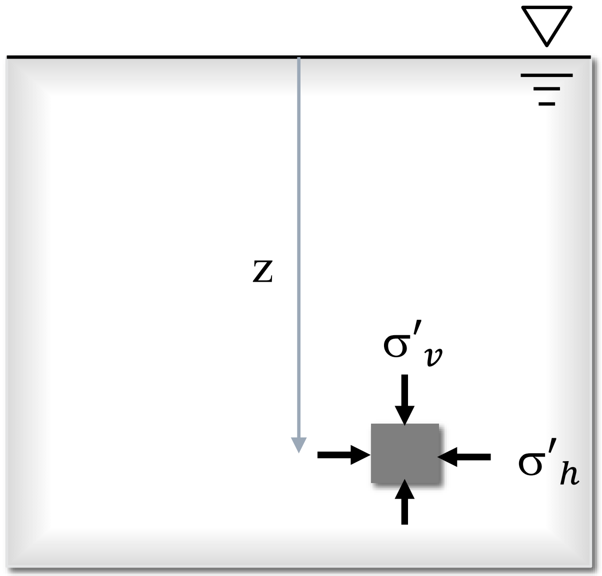
The Rankine's theory (1857) considers the soil to be in a state of *plastic equilibrium*: condition where each point in the soil mass is on the verge of failure.

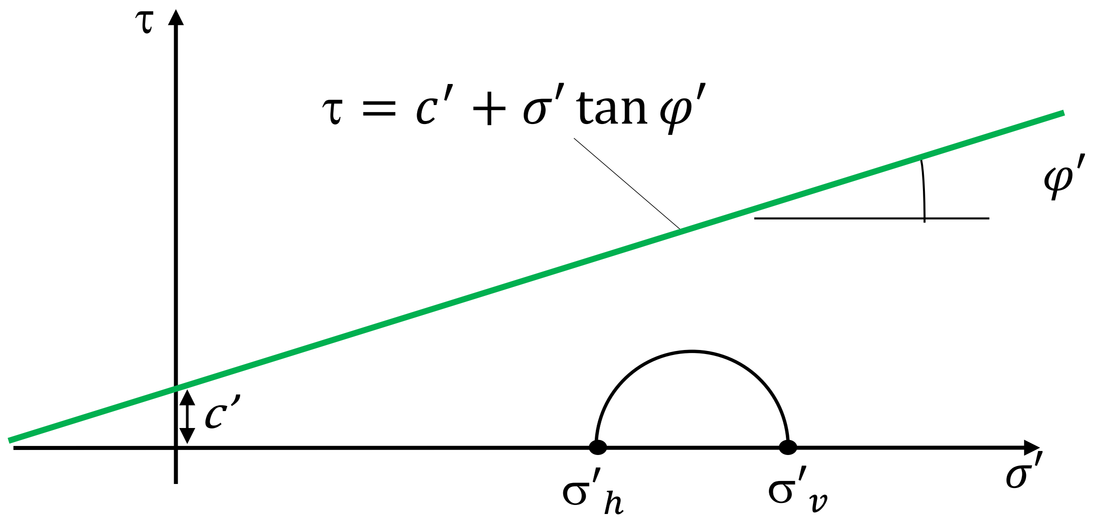
He considered the following assumptions:

- the soil is homogeneous, isotropic and has an internal friction;
- the ground level is horizontal;
- soil-structure friction equal to zero: shear stresses are zero (at τ=0); thus the vertical and horizontal stresses are yet principal stress directions.

Rankine's theory

In situ stress distribution:





Total stress

$$\sigma_v = \gamma_{sat} z$$

$$\sigma_h = K_0\, \gamma' z + \gamma_w z$$

Pore water pressure distribution

$$p_w = \gamma_w z$$

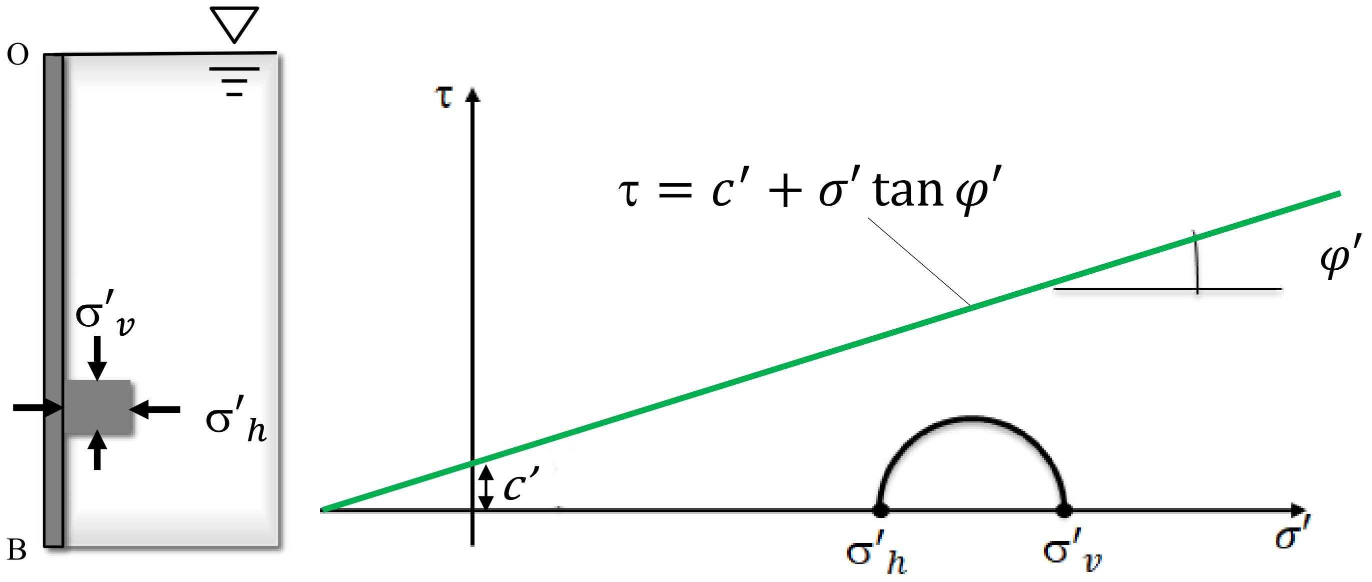
Effective stress

$$\sigma'_v = z\gamma_{sat} - \gamma_w z = \gamma' z$$

$$\sigma'_h = K_0\, \gamma' z$$

Rankine's theory

Let's insert a smooth rigid infinite metal plate, removing the left part of the soil:



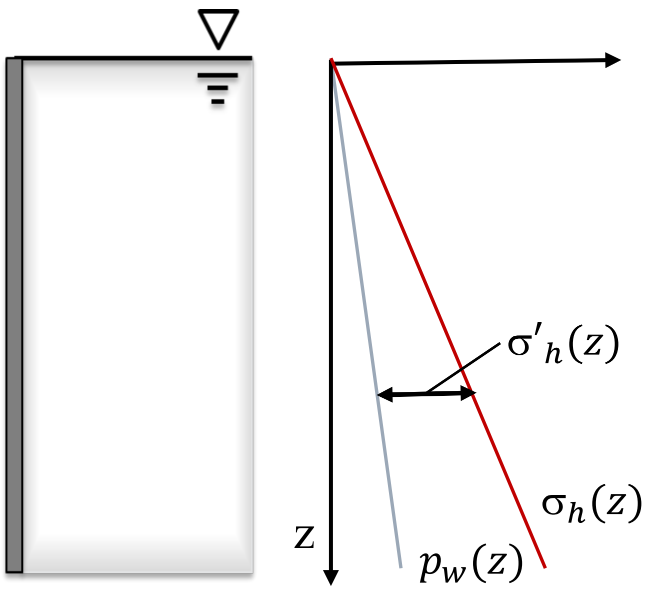
At rest, lateral earth pressure:

Total stress

$$\sigma_h = K_0\, \gamma' z + \gamma_w z$$

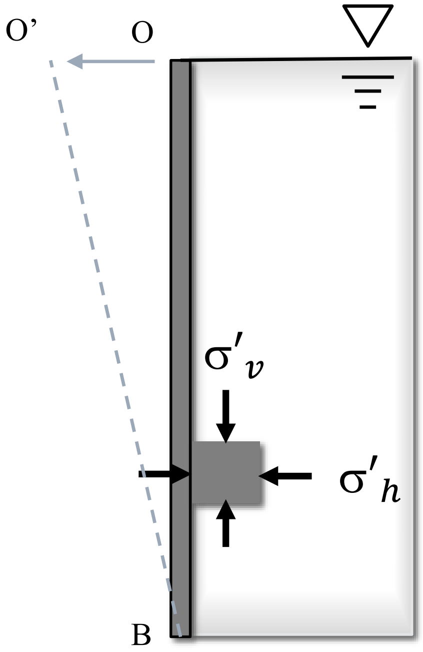
Effective stress

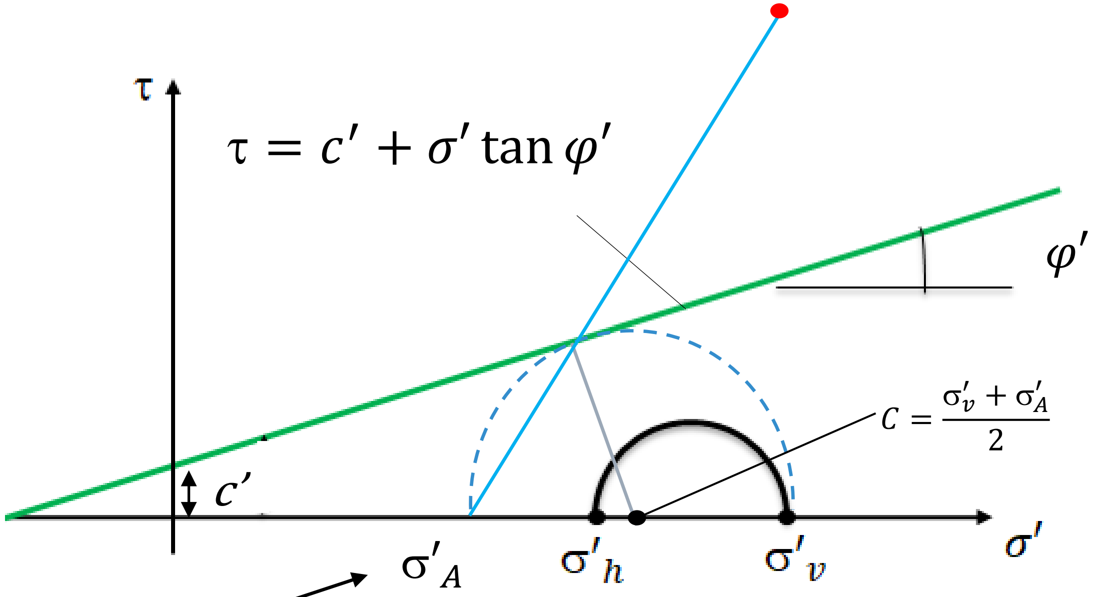
$$\sigma'_h = K_0\, \gamma' z$$

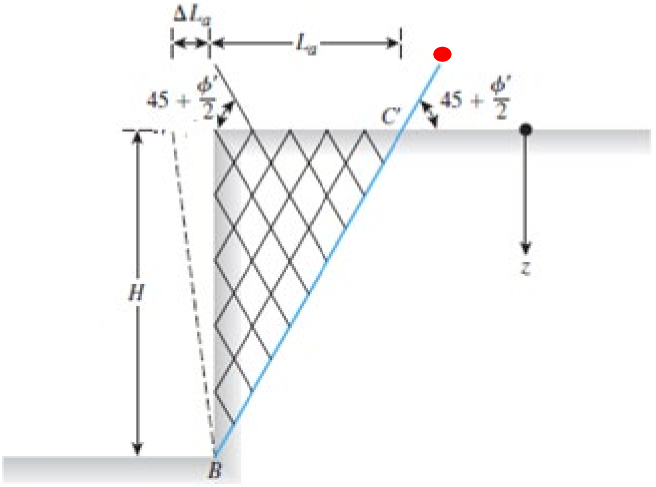


Rankine's theory

Active state:



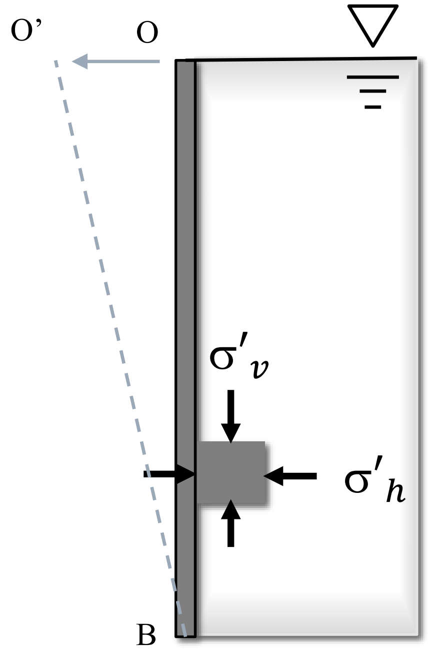


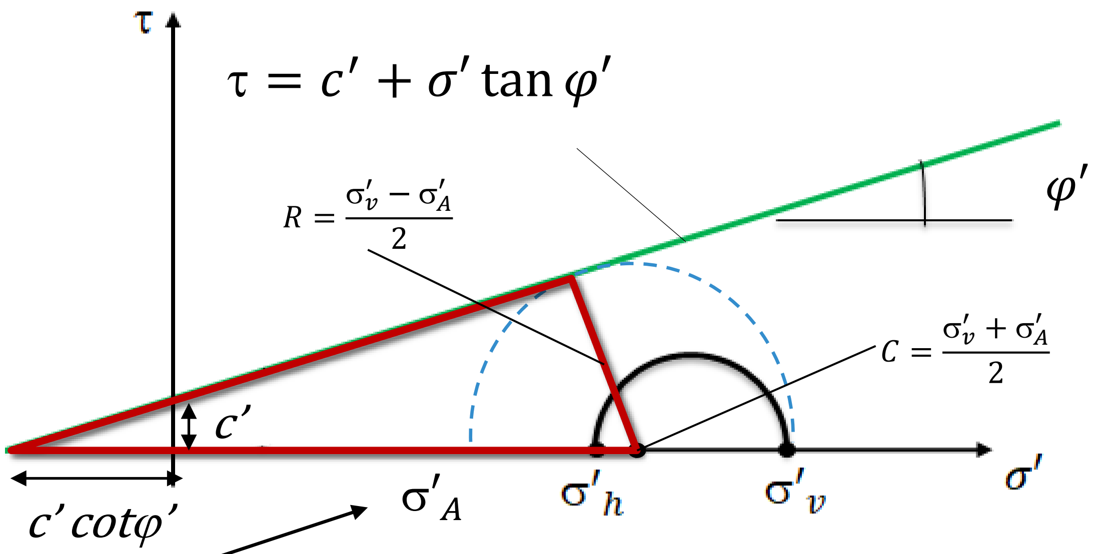


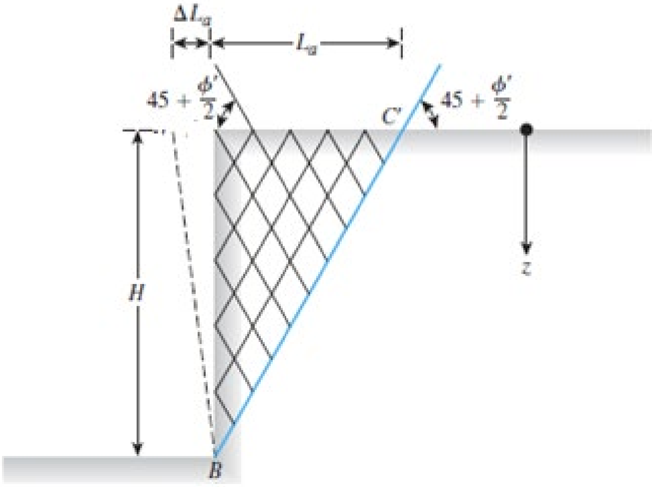
Active Lateral Earth pressure:

Rankine's theory

Active state:







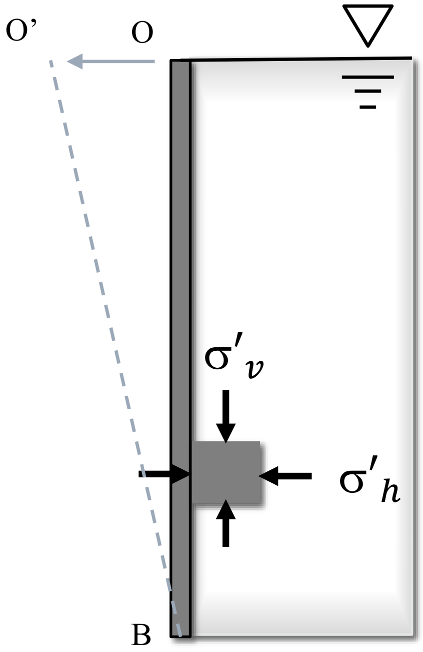
Active Lateral Earth pressure:

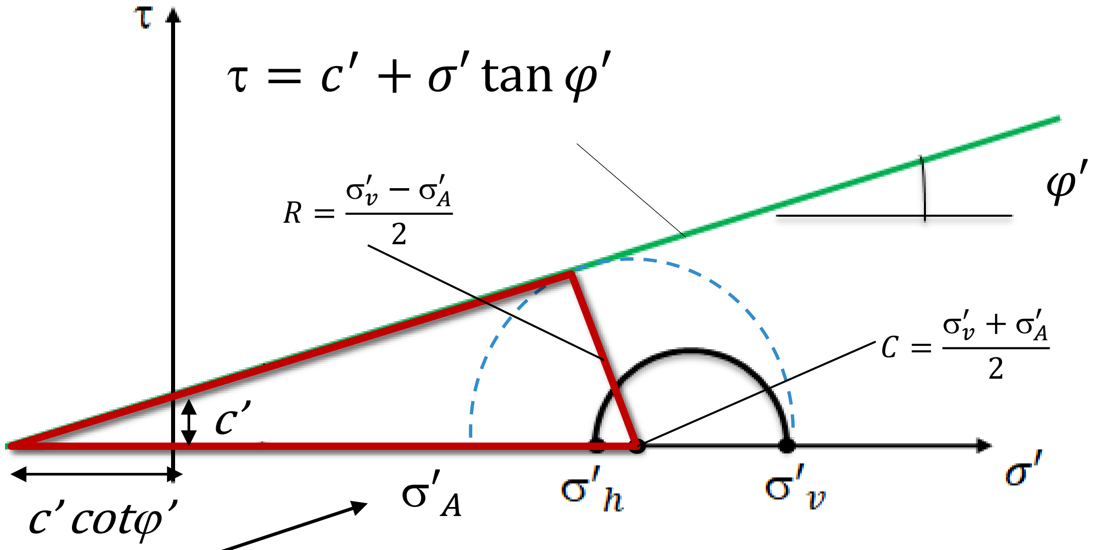
$$\frac{\sigma'_v - \sigma'_A}{2} = \left(\frac{\sigma'_v + \sigma'_A}{2} + c' \cot \varphi'\right) \sin \varphi'$$

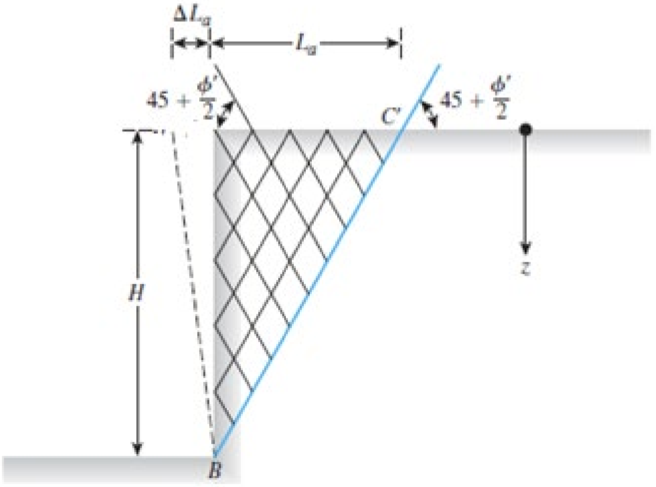
$$\sigma'_A = \sigma'_v \left(\frac{1 - \sin \varphi'}{1 + \sin \varphi'}\right) - 2c' \left(\frac{\cos \varphi'}{1 + \sin \varphi'}\right) = \sigma'_v \underbrace{tan^2\left(\frac{\pi}{4} - \frac{\varphi'}{2}\right)}_{K_A} - 2c' \underbrace{tan\left(\frac{\pi}{4} - \frac{\varphi'}{2}\right)}_{\sqrt{K_A}}$$

Rankine's theory

Active state:







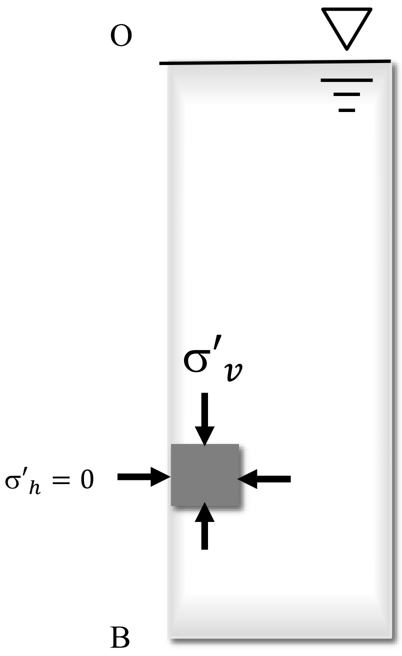
Active Lateral Earth pressure:

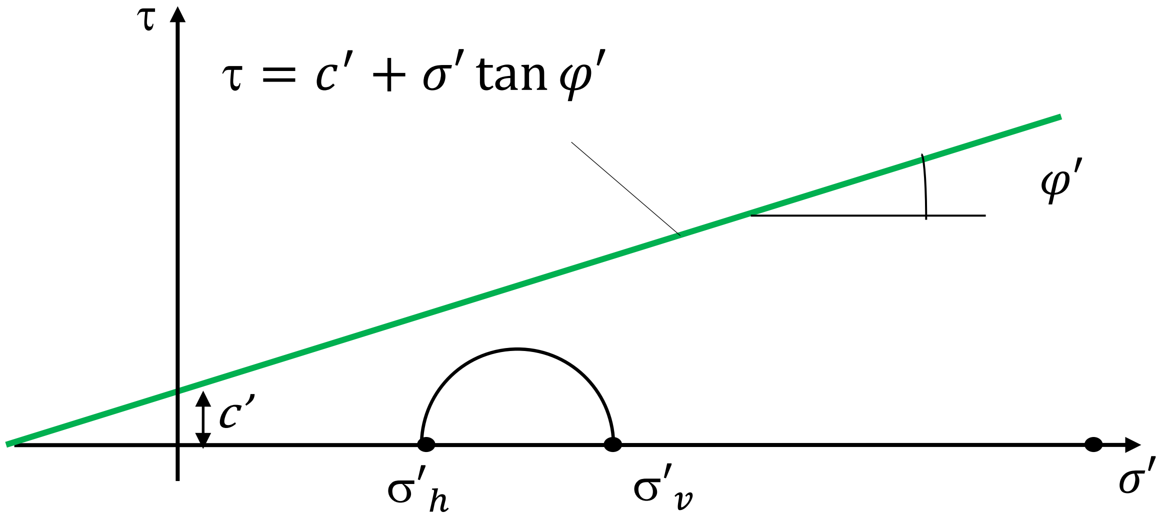
Effective stress: $\sigma'_A = K_A \gamma' z - 2c'\sqrt{K_A}$

Total stress: $\sigma_A = K_A \gamma' z - 2c'\sqrt{K_A} + \gamma_w z$

Rankine's theory

Passive state:

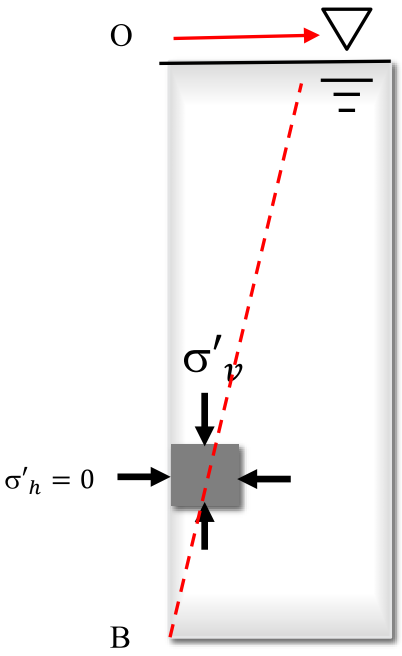


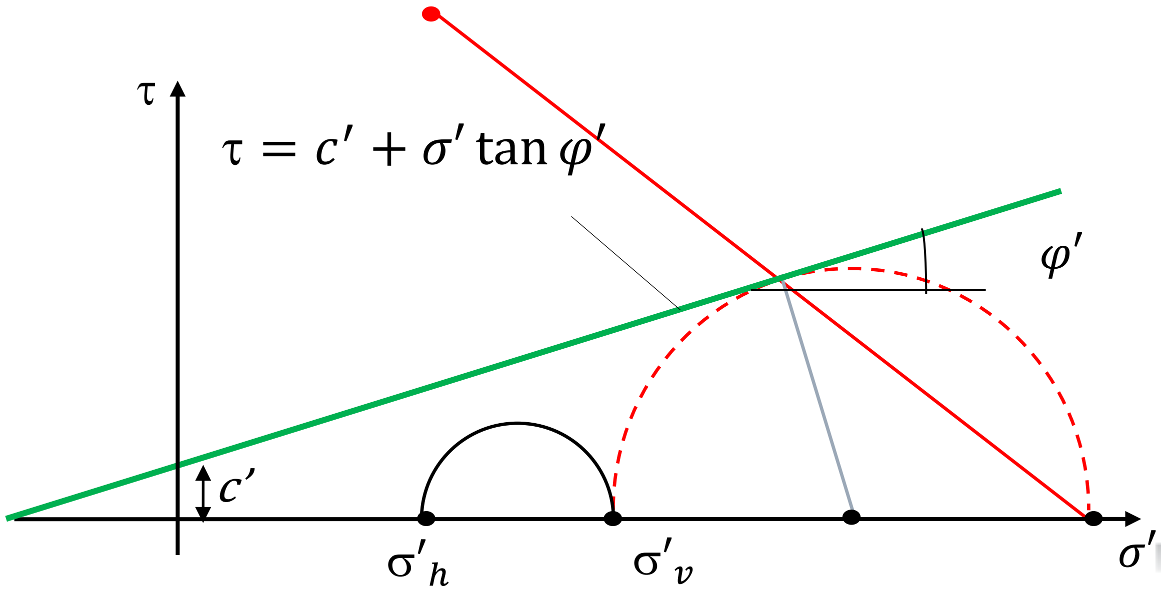


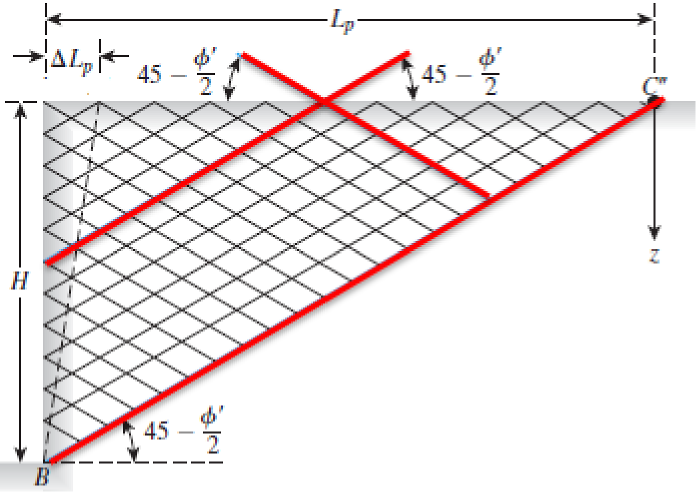
Passive Lateral Earth pressure:

Rankine’s theory

Passive state:



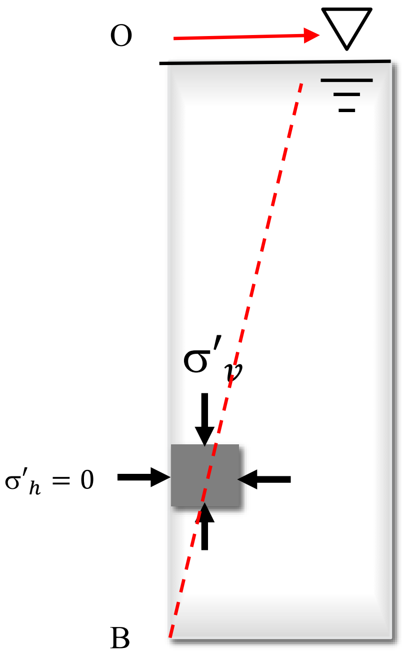


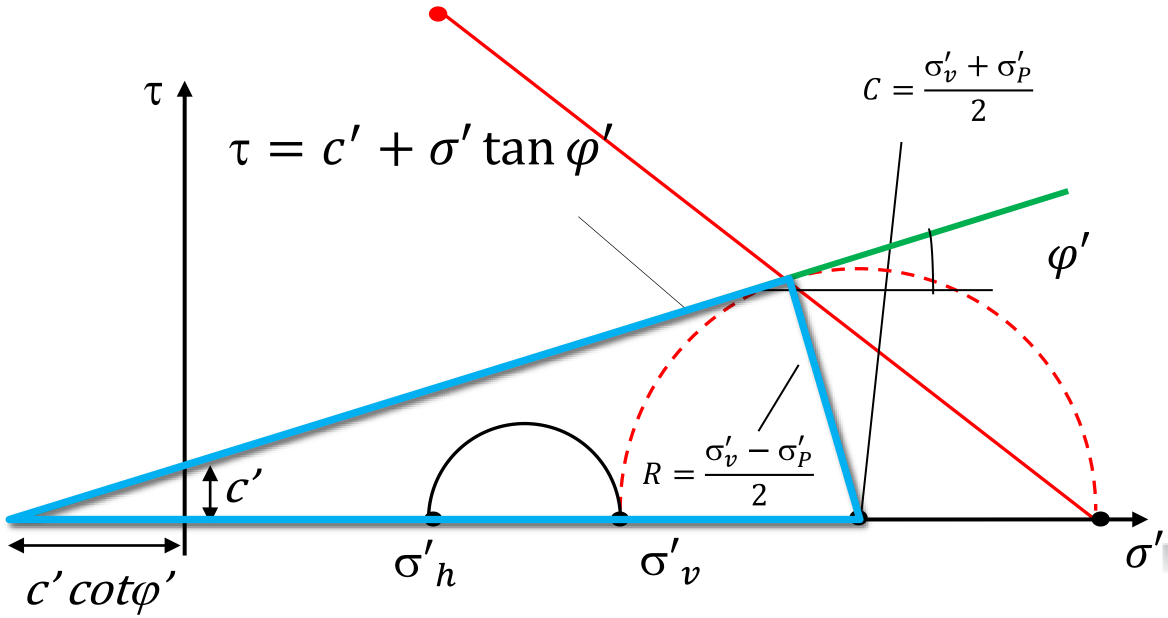


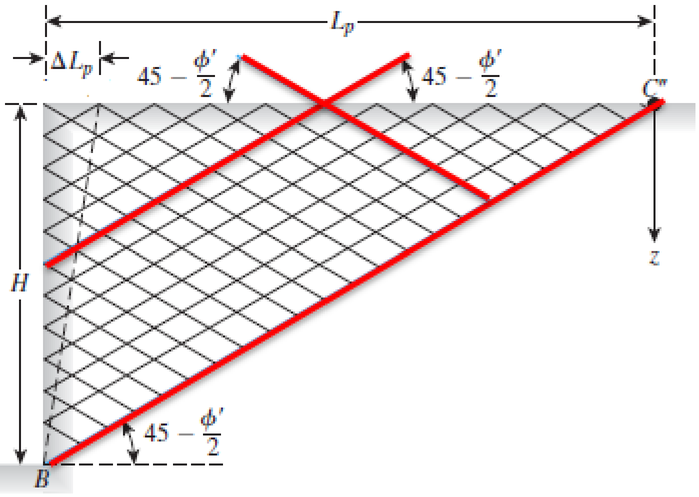
Passive Lateral Earth pressure:

Rankine's theory

Passive state:







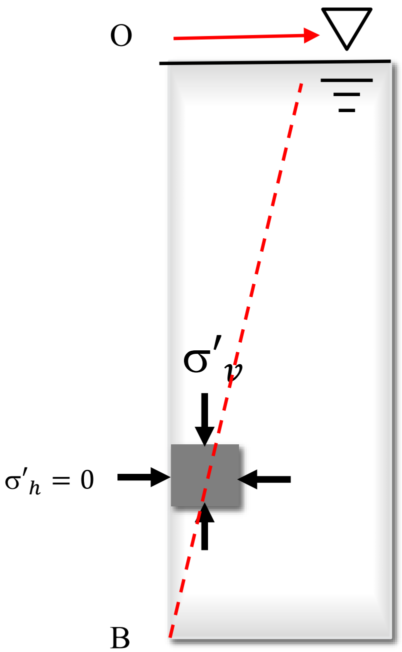
Passive Lateral Earth pressure:

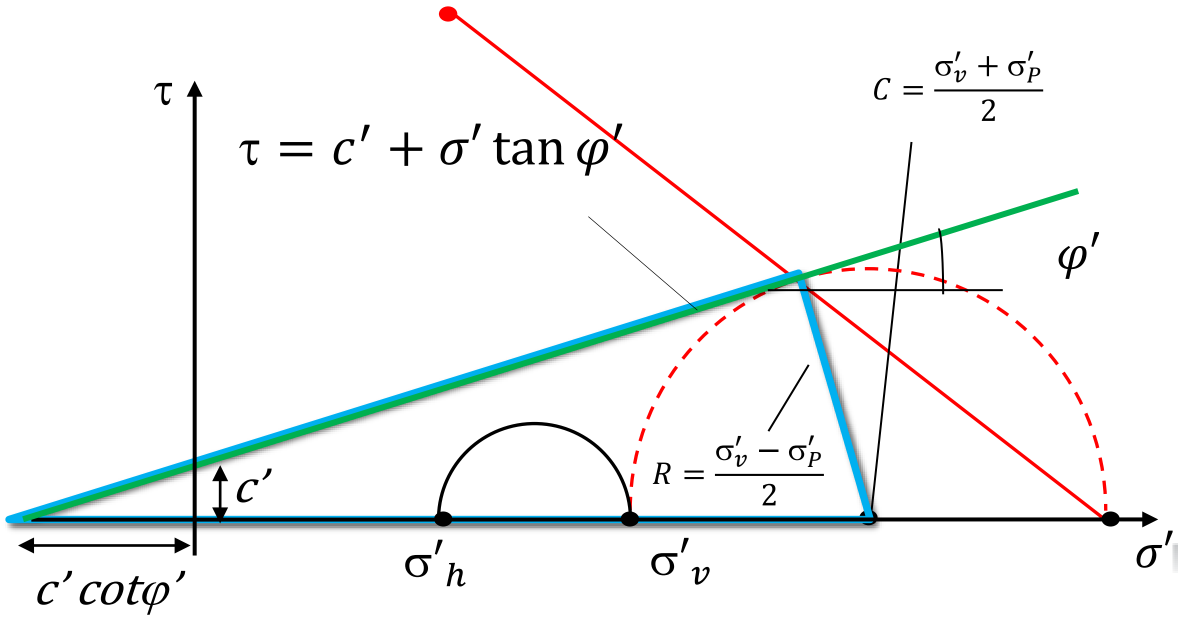
$$\frac{\sigma'_P - \sigma'_v}{2} = \left(\frac{\sigma'_v + \sigma'_P}{2} + c' \cot \varphi'\right) \sin \varphi'$$

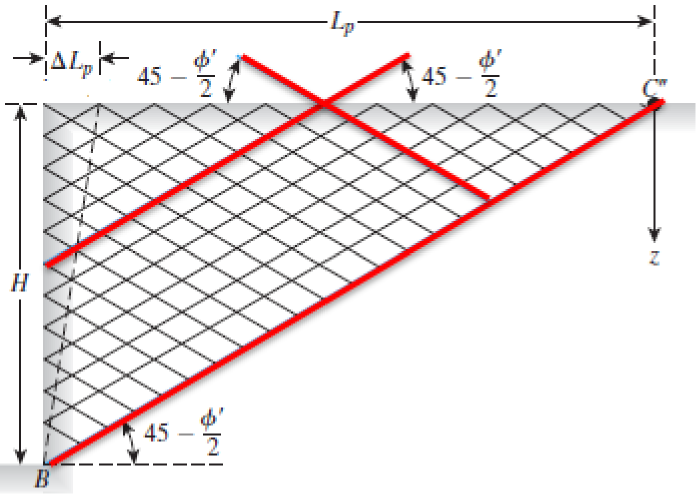
$$\sigma'_P = \sigma'_v \underbrace{tan^2\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)}_{K_P} + 2c' \underbrace{tan\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)}_{\sqrt{K_P}}$$

Rankine's theory

Passive state:







Passive Lateral Earth pressure:

Effective stress: $\sigma'_P = K_P\,\gamma' z + 2c'\sqrt{K_P}$

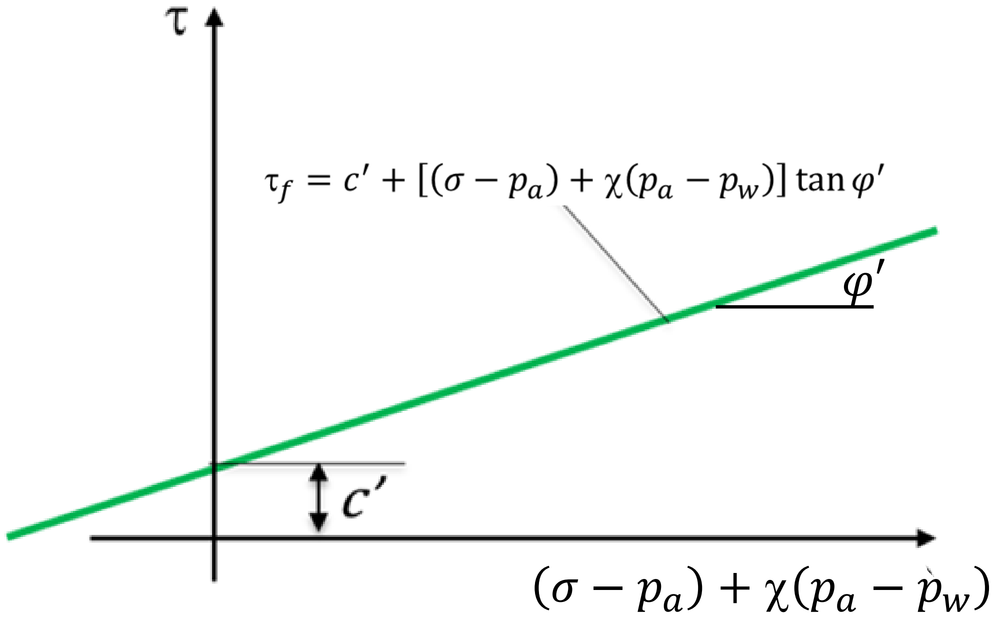
Total stress: $\sigma_P = K_P \gamma' z + 2c'\sqrt{K_P} + \gamma_w z$

Shear strength of Unsaturated Soils

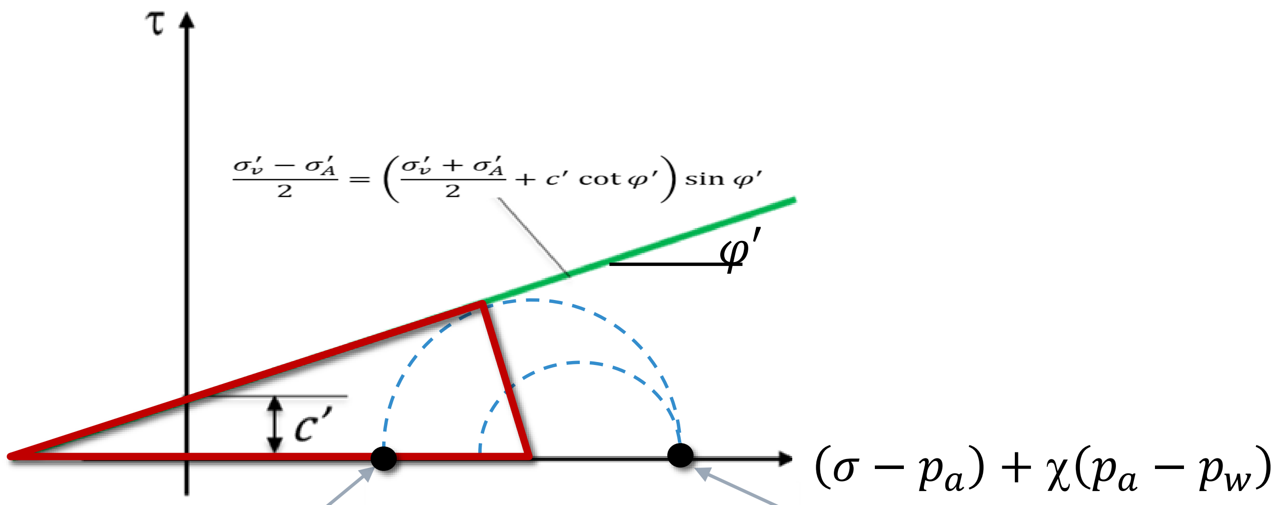
The shear strength of Unsaturated soils is function of two effective shear strength parameters (c' and φ') and a single stress variable, σ' (adopting the **Bishop's definition of effective stress**):

$$\tau_f = c' + [(\sigma - p_a) + \chi(p_a - p_w)] \tan \varphi'$$

χ Is the effective stress parameter usually imposed equal to the degree of saturation S_r



Rankine's theory for Unsaturated Soils



$\sigma'_A = (\sigma_A - p_a) + \chi(p_a - p_w)$ $\quad \sigma'_v = (\sigma_v - p_a) + \chi(p_a - p_w)$

Active state:

$$\frac{\sigma'_v - \sigma'_A}{2} = \left(\frac{\sigma'_v + \sigma'_A}{2} + c' \cot\varphi'\right)\sin\varphi'$$

$$\sigma'_A = \sigma'_v\left(\frac{1 - \sin\varphi'}{1 + \sin\varphi'}\right) - 2c'\left(\frac{\cos\varphi'}{1 + \sin\varphi'}\right) = \sigma'_v K_A - 2c'\sqrt{K_A}$$

Substituting the Bishop's definition of effective stress:

$\chi = S_r$

$$(\sigma_A - p_a) + \chi(p_a - p_w) = [(\sigma_v - p_a) + \chi(p_a - p_w)]K_A - 2c'\sqrt{K_A}$$

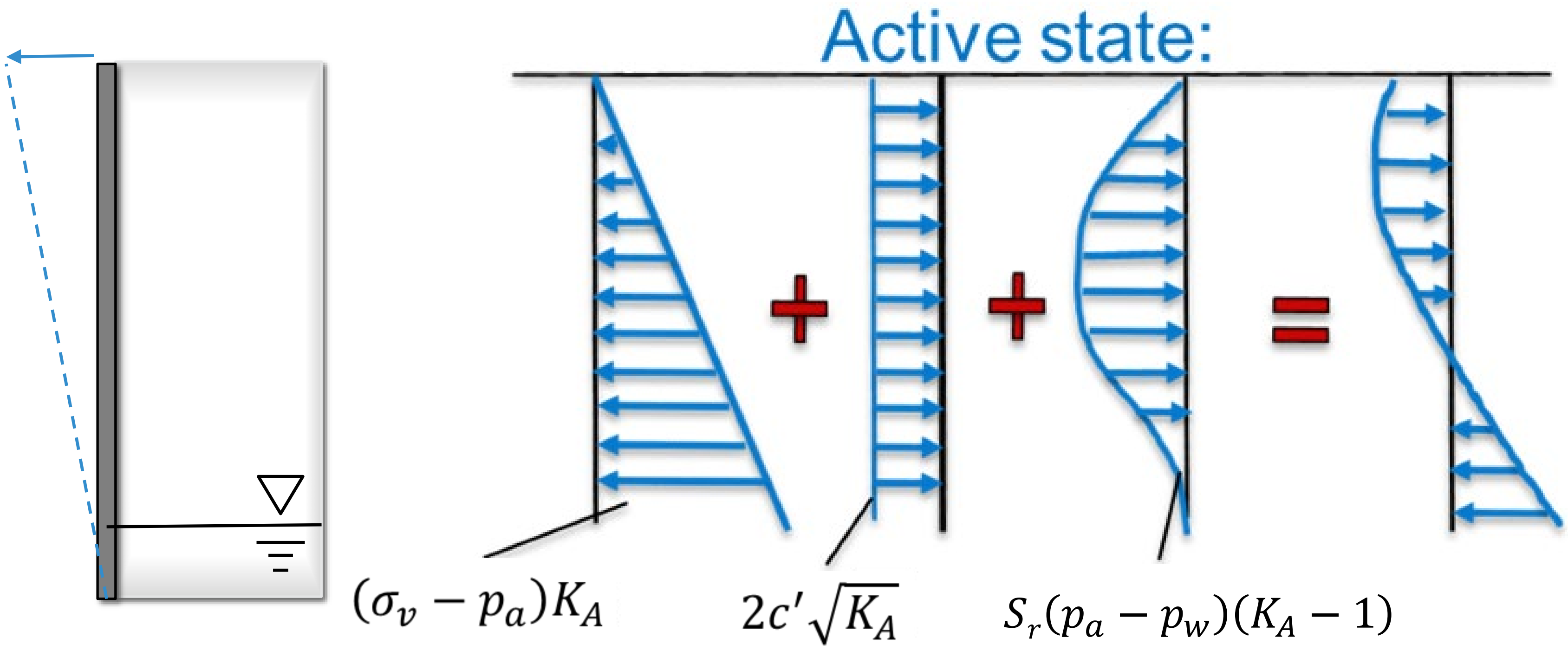
$$\sigma_A - p_a = (\sigma_v - p_a)K_A - 2c'\sqrt{K_A} + S_r(p_a - p_w)(K_A - 1)$$

Rankine's theory for Unsaturated Soils

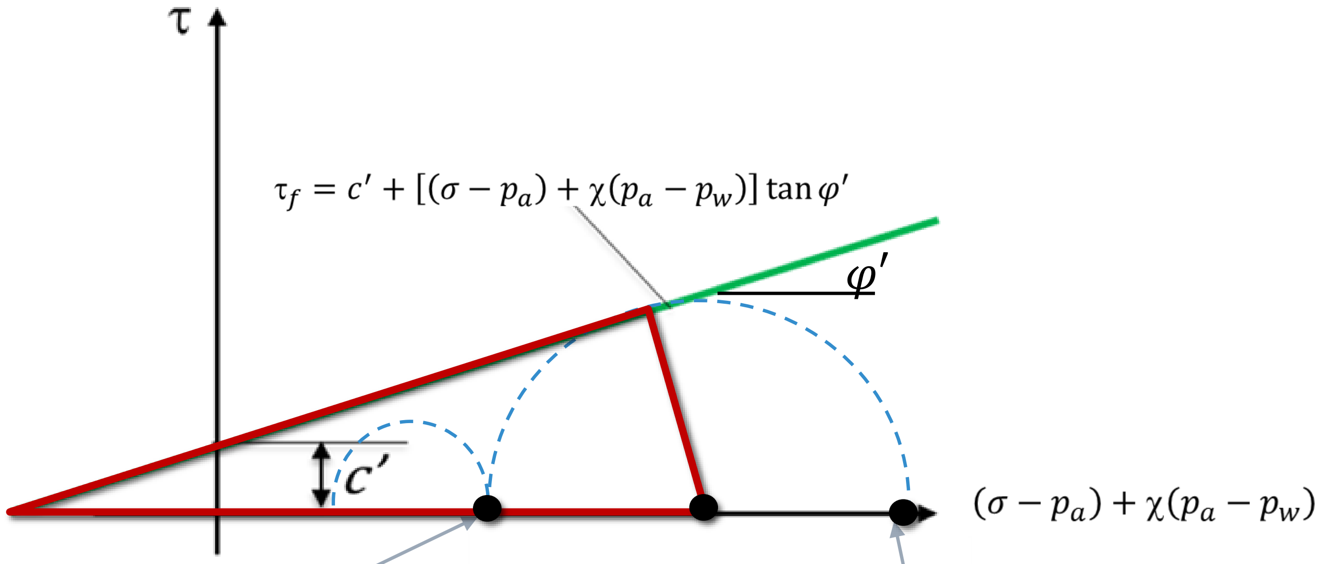
The Rankine's theory can be extended to Unsaturated Soils for active case:

$$\sigma_A - p_a = (\sigma_v - p_a)K_A - 2c'\sqrt{K_A} + S_r(p_a - p_w)(K_A - 1)$$

$\sigma_v = \gamma_{sat} z$ And usually $p_a = 0$



Shear strength of Unsaturated Soils



$\sigma'_v = (\sigma_v - p_a) + \chi(p_a - p_w)$ $\sigma'_P = (\sigma_P - p_a) + \chi(p_a - p_w)$

Passive state:

$$\frac{\sigma'_P - \sigma'_v}{2} = \left(\frac{\sigma'_v + \sigma'_P}{2} + c'\cot\varphi'\right)\sin\varphi'$$

$$\sigma'_P = \sigma'_v\left(\frac{1+\sin\varphi'}{1-\sin\varphi'}\right) + 2c'\left(\frac{\cos\varphi'}{1-\sin\varphi'}\right) = \sigma'_v K_P + 2c'\sqrt{K_P}$$

Substituting the Bishop's definition of effective stress:

$\chi = S_r$

$$(\sigma_P - p_a) + \chi(p_a - p_w) = [(\sigma_v - p_a) + \chi(p_a - p_w)]K_P + 2c'\sqrt{K_P}$$

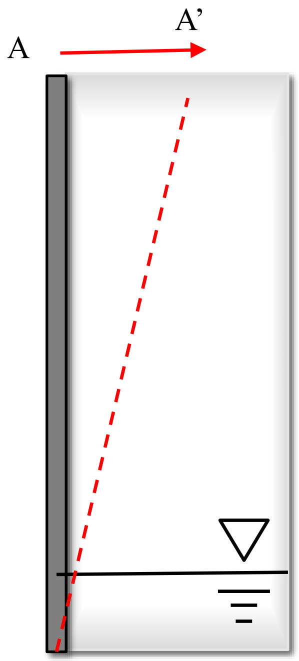
$$\sigma_P - p_a = (\sigma_v - p_a)K_P + 2c'\sqrt{K_P} + S_r(p_a - p_w)(K_P - 1)$$

Rankine's theory for Unsaturated Soils

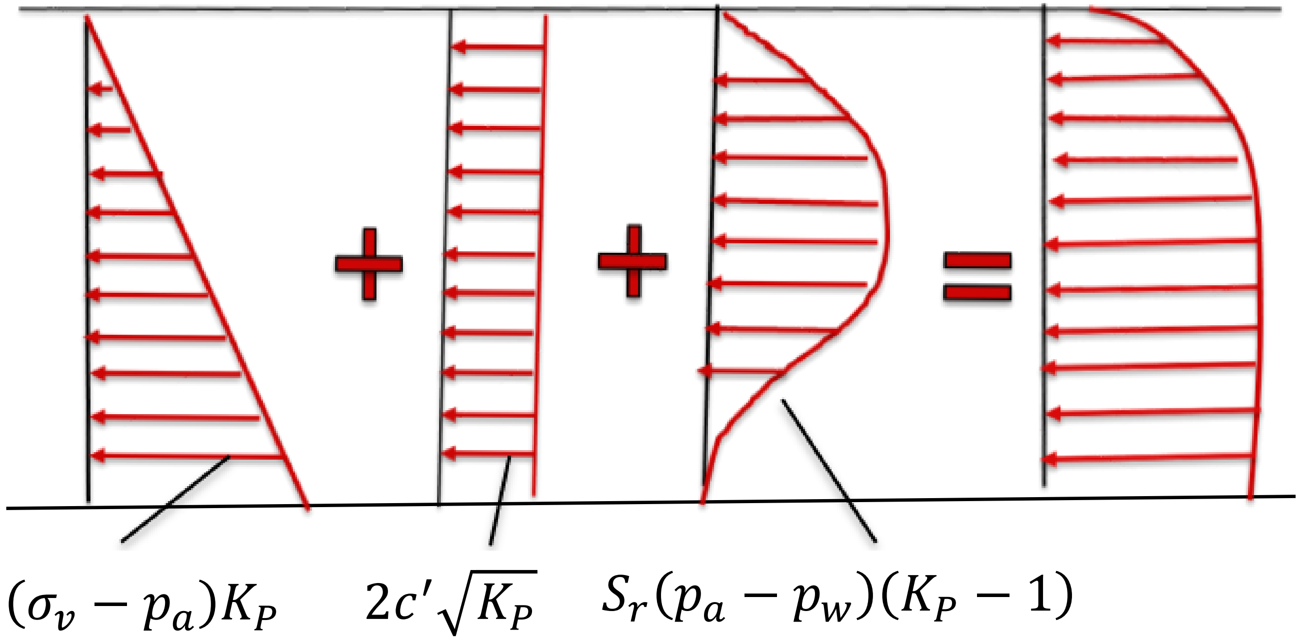
The Rankine's theory can be extended to Unsaturated Soils for passive case:

$$\sigma_P - p_a = (\sigma_v - p_a)K_P + 2c'\sqrt{K_P} + S_r(p_a - p_w)(K_P - 1)$$

$\sigma_v = \gamma_{sat} z$ And usually $p_a = 0$

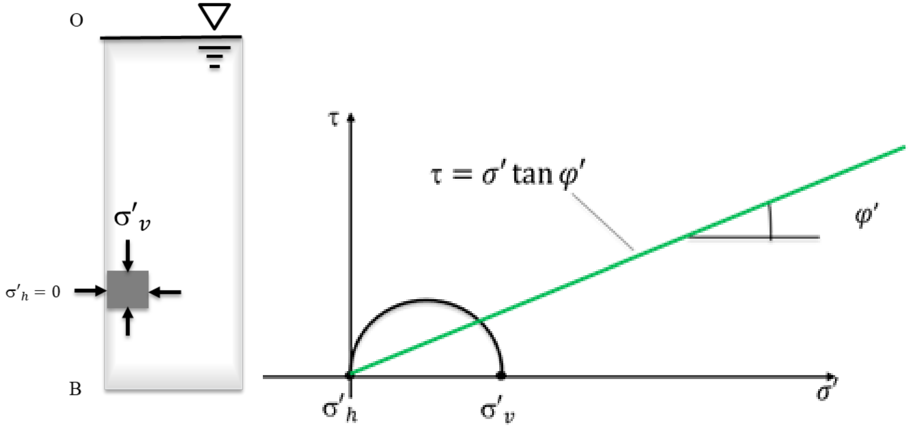


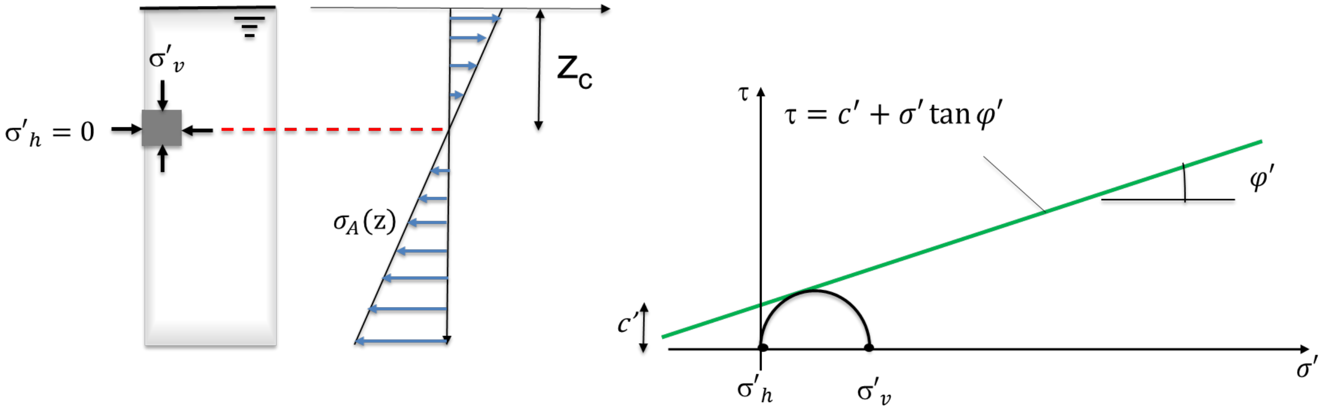
Passive state:



Vertical trench - Critical height

Saturated Soils





- **Non cohesive soils:**

  Any REV on the excavation face is subjected to a stress state not compatible with the failure condition.
  The vertical trench cannot be self-supported.

- **Cohesive soils:**

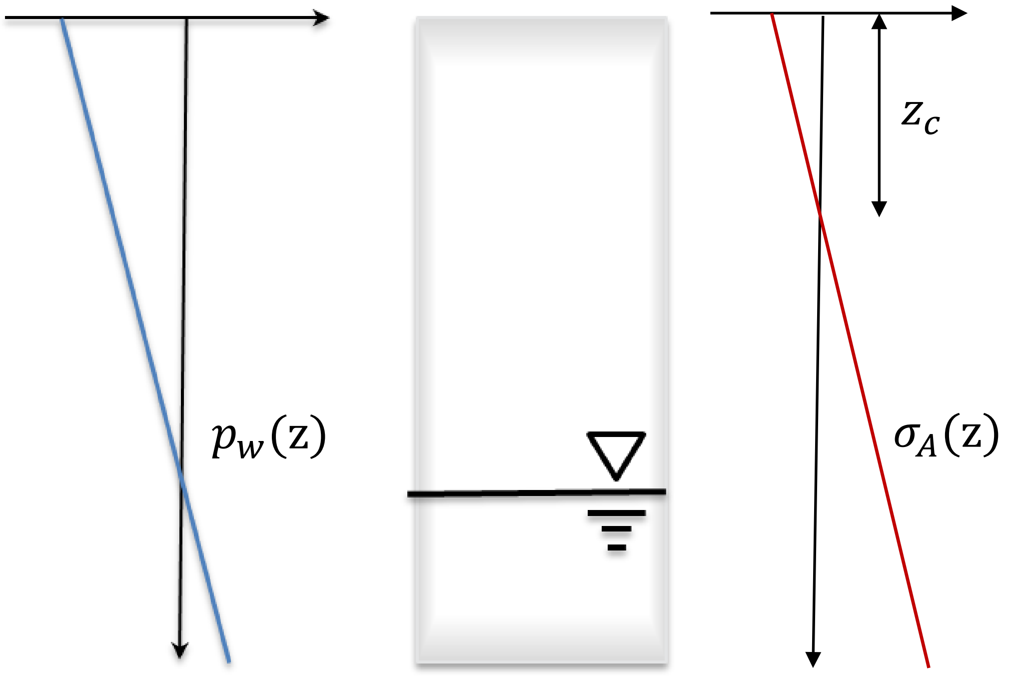
  Drained condition

$$\sigma'_A = K_A\, \gamma' z - 2c'\sqrt{K_A} = 0$$

$$z_c = \frac{2c'}{\gamma'\sqrt{K_A}}$$

Vertical trench - Critical height

Unsaturated Soils



Distribution of matric suction ?

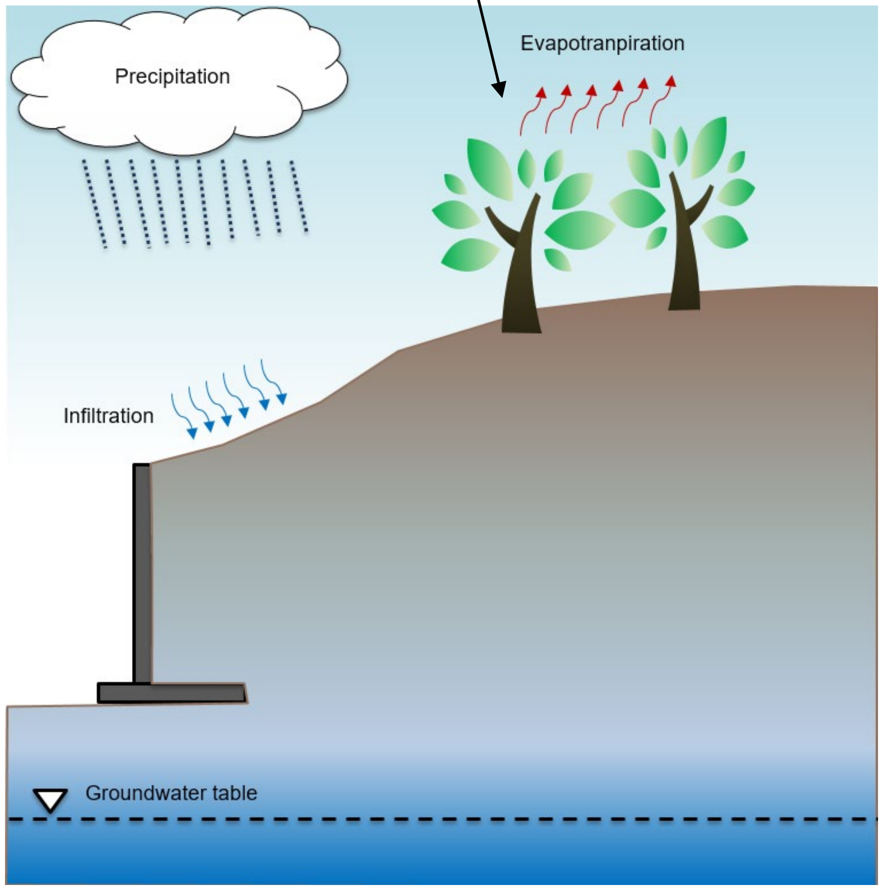
$$z_c = \frac{2c'}{\gamma\sqrt{K_A}} + \frac{\chi(p_a - p_w)}{\gamma}\left(\frac{1}{K_A} - 1\right)$$

$$\sigma_A - p_a = (\sigma_v - p_a)K_A - 2c'\sqrt{K_A} + \chi(p_a - p_w)(K_A - 1) = 0$$

$\gamma = f(S_r)$ To be in the safe side we can impose $\gamma = \gamma_{sat}$

Effect of infiltrations on the lateral earth pressure

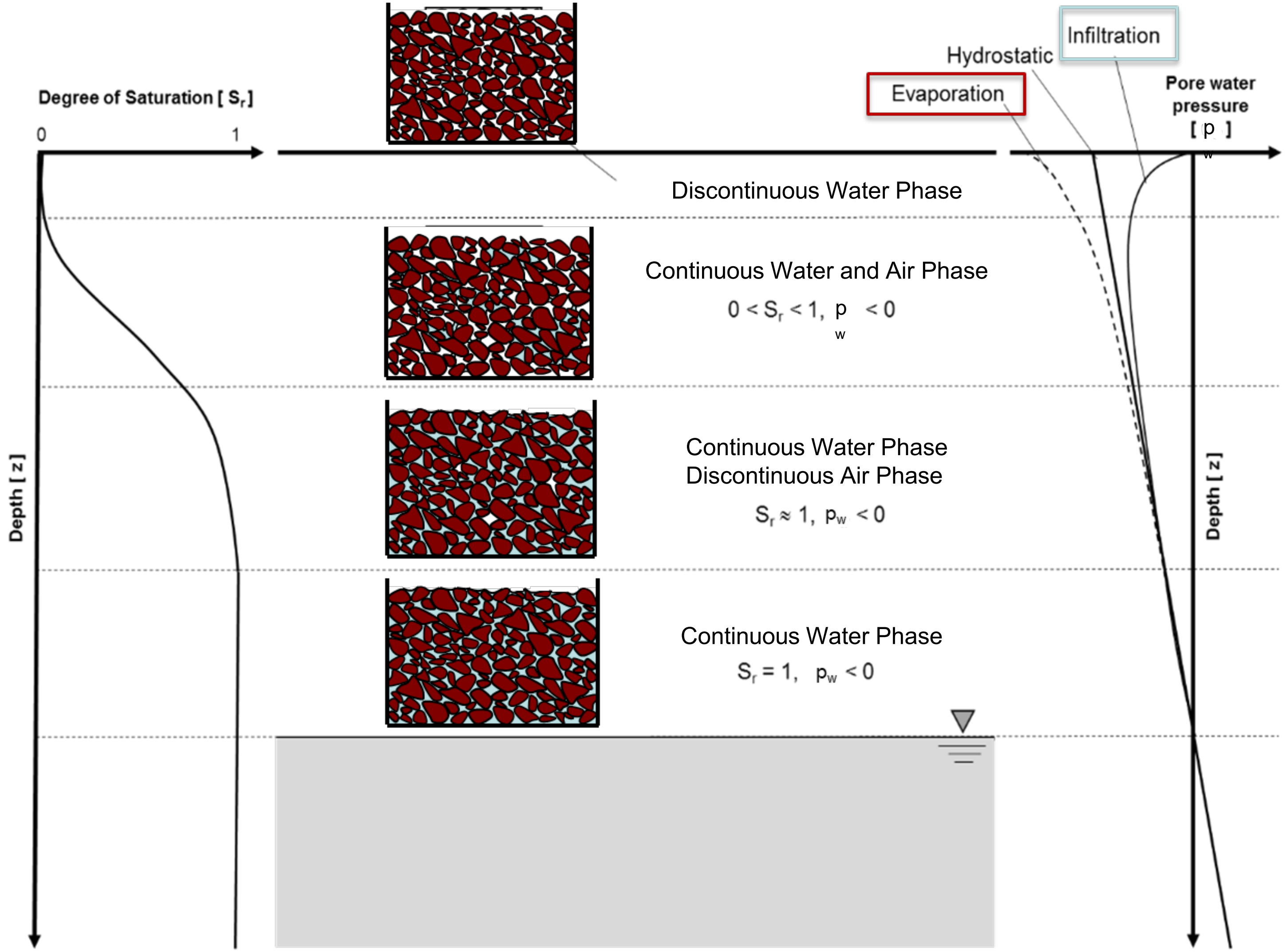
Interaction between retaining structures and environmental actions



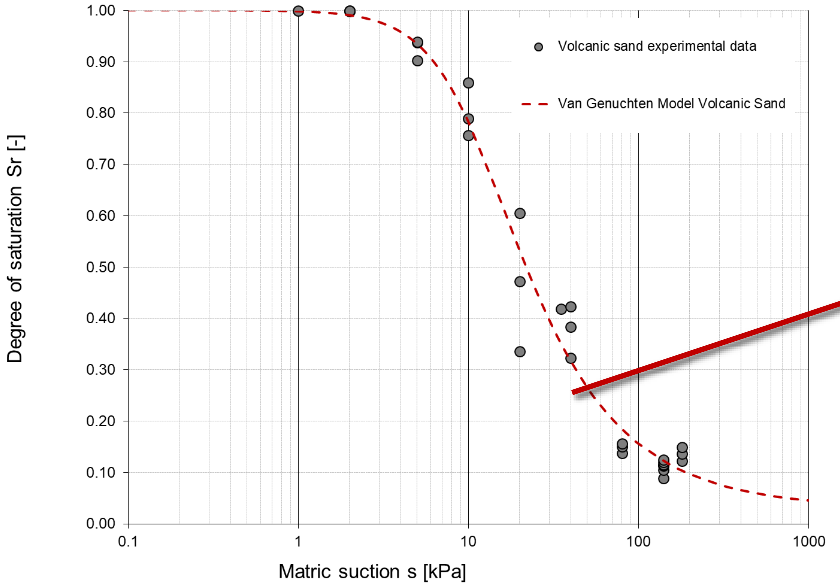
A "practical" remediation to limit infiltrations during a retaining wall construction

- What happens to the matric suction during the rainfall events?
- How to model the hydraulic behaviour of unsaturated soils?

Evolution of saturation degree and matric suction with depth



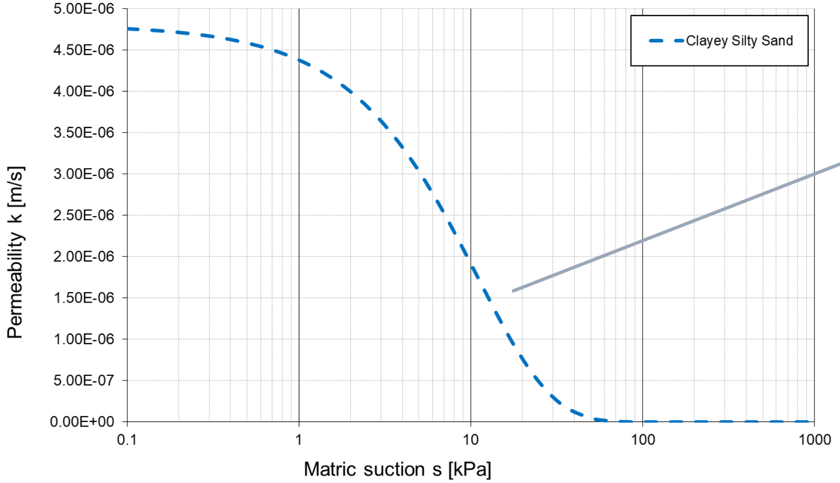
Hydraulic behaviour of geomaterials



Van Genuchten, M., 1980

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

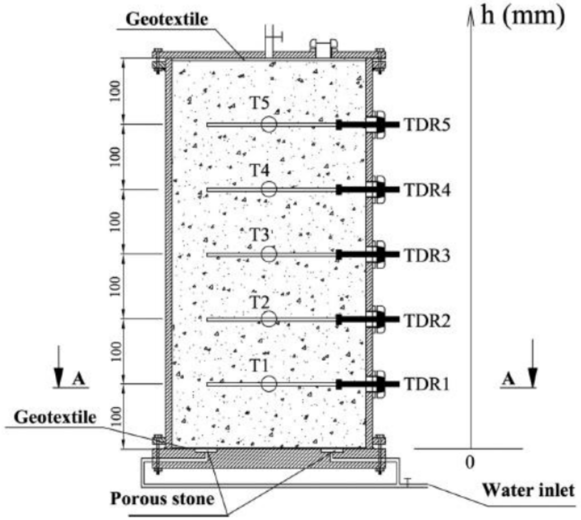
α ,n, m calibration parameters



Gardner's model

$$k = k_s e^{-\alpha(p_a - p_w)}$$

Infiltration



Duong et al., 2014

Darcy law for saturated and unsaturated soils

Darcy's law for saturated soils:

$$Q = -k_s A \left[\frac{dh}{dz}\right] \longrightarrow q = \frac{Q}{A} = -k_s \left[\frac{dh}{dz}\right]$$

Extended Darcy's law for unsaturated soils (steady state):

$$h = z + h_m = z + \frac{p_a - p_w}{\rho_w g}$$

$$q = -k \left[\frac{d\left(z + \frac{p_a - p_w}{\rho_w g}\right)}{dz}\right]$$

$$q = -k \left[\frac{d(p_a - p_w)}{\rho_w g dz} + 1\right]$$

h = hydraulic head [L]

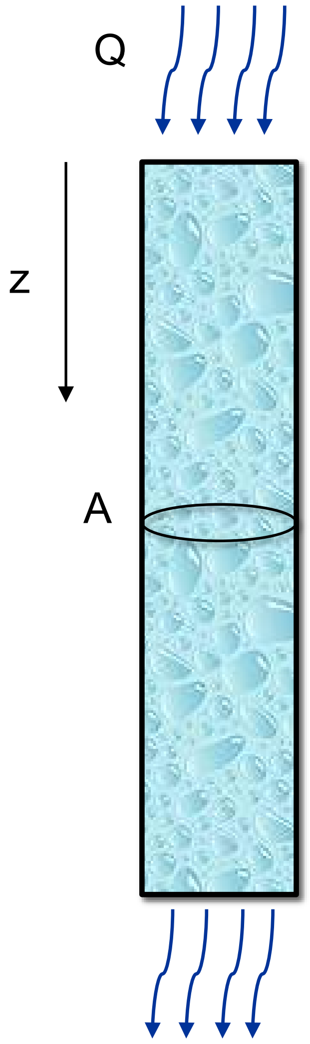
Q = discharge rate [L^3 T^{-1}]

A = Cross section [L^2]

q = specific discharge rate [L T^{-1}]

k= $f(p_a - p_w)$= unsaturated hydraulicconductivity [L T^{-1}]

k_s= saturated hydraulic conductivity [L T^{-1}]



Analytical solution for Matric suction profiles

Hp: Steady State and Mono-dimensional infiltration / evaporation rate

Coupling the Darcy' s law with the Gardner's model and integrating over depth:

$$\begin{cases} q = -k\left[\dfrac{d(p_a - p_w)}{\gamma_w dz} + 1\right] \\ k = k_s e^{[-\alpha(p_a - p_w)]} \end{cases}$$

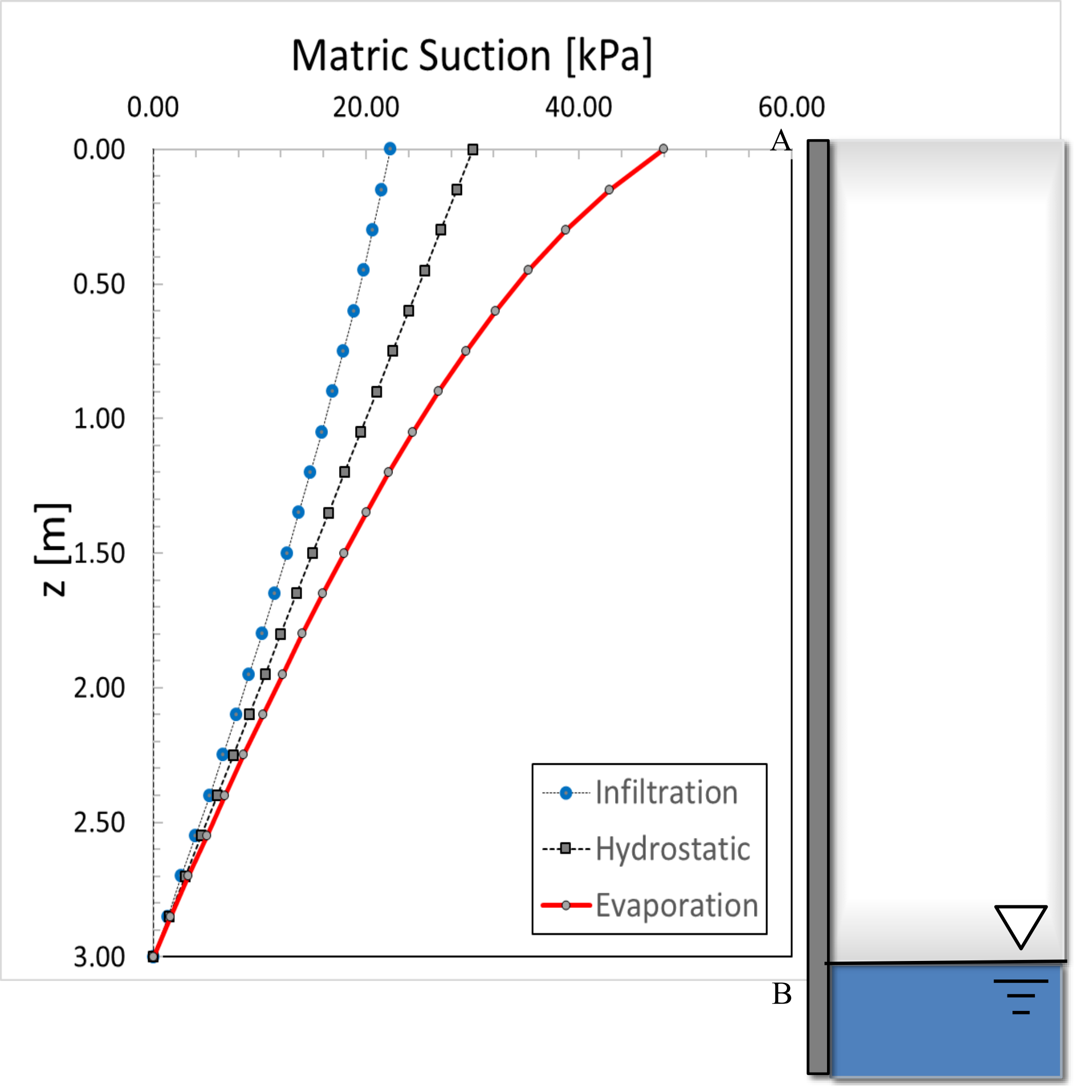
k= unsaturated hydraulic conductivity

k_s= saturated hydraulic conductivity

It is possible to achieve the distribution of matric suction over depth:

$$(p_a - p_w) = -\frac{1}{\alpha}\ln\left[\left(1 + \frac{q}{k_s}\right)e^{-\gamma_w \alpha z} - \frac{q}{k_s}\right]$$

q = specific discharge rate equal to infiltration or evaporation rate



Summary

Definition of effective stress:

$$\sigma' = \sigma_{net} + S_r\, s \qquad (\sigma_{net} = \sigma - p_a)$$

$$s = (p_a - p_w)$$

SWRC and effective stress parameter:

$$\chi \approx S_r = \left\{\frac{1}{1 + [\alpha(p_a - p_w)]^n}\right\}^m$$

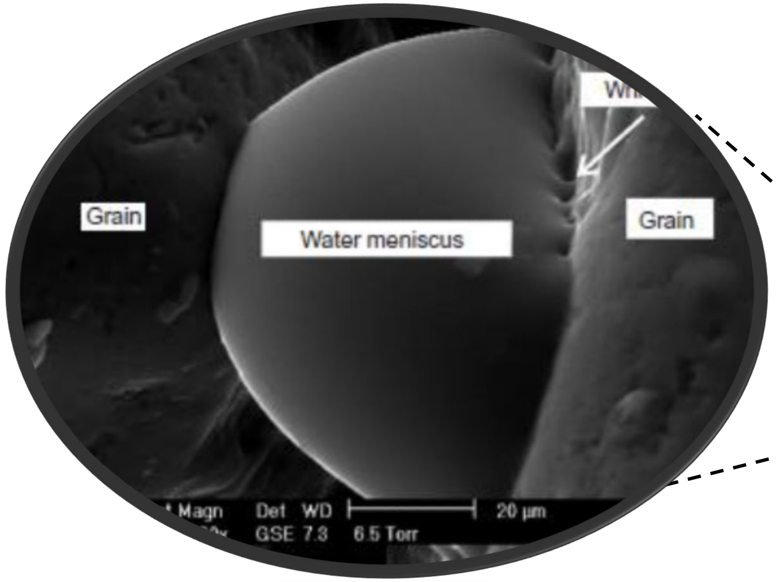
Mohr Coulomb failure criterion for Unsaturated soils:

$$\tau_f = c' + \left\{\sigma_{net} + \left\{\frac{1}{1 + [\alpha(p_a - p_w)]^n}\right\}^m (p_a - p_w)\right\} \tan\varphi'$$

Distribution of matric suction above GWT at different infiltration rates:

$$(p_a - p_w) = -\frac{1}{\alpha}\ln\left[\left(1 + \frac{q}{k_s}\right)e^{-\gamma_w \alpha z} - \frac{q}{k_s}\right]$$

Hp: Monodimensional infiltration with continuity of the water phase



Lateral Earth Pressure at active/passive state in Unsaturated Soils with infiltration/evaporation:

$$\sigma_{A/P} - p_a = (\sigma_v - p_a)K_{A/P} \mp 2c'\sqrt{K_{A/P}} + \left\{\frac{1}{1 + [\alpha(p_a - p_w)]^n}\right\}^m \left\{-\frac{1}{\alpha}\ln\left[\left(1 + \frac{q}{k_s}\right)e^{-\gamma_w \alpha z} - \frac{q}{k_s}\right]\right\}(K_{A/P} - 1)$$

Conclusion

- Retaining walls usually retain soils that are in unsaturated conditions
- The presence of suction can modify significantly the Lateral Earth Pressure
- Currently design procedures take into account either dry or saturated conditions
- To be on the safe side according to SIA261 and Eurocode 7: For structures retaining soils of medium or low permeability (silts and clays), water pressures shall be assumed to act behind the wall; unless a reliable drainage system is installed, or infiltration is prevented, the values of water pressures shall correspond to a water table at the top of the layer characterized by a low permeability.